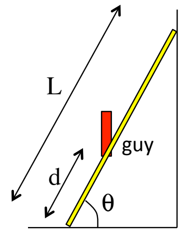
Problem 12.13

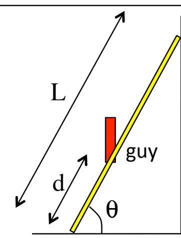


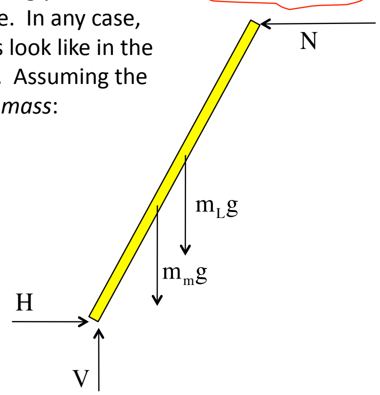
This is a classic "rigid body" problem. There will usually be three unknown forces acting in such problems. Because the wall is assumed to be frictionless, there is no vertical force component acting at the wall, just a horizontal "normal" component. At the floor, there is a normal force due to the floor upward and a frictional force in the horizontal (oddly, and your teacher may or may not explain why this is the case, but those forces are usually identified as "H" for the horizontal frictional force and "V" for the vertical normal force—kindly noticed that that was what they are referred to as in the problem). In any case, to solve for the three unknowns we need three equations which we can get from summing forces in two directions and summing torques about any point we'd like. All the sums will equal zero as there is no *acceleration* or *angular acceleration* in the system.

With that preamble, we proceed.

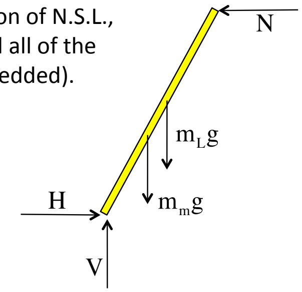
a.) What are the forces acting at the floor?

Note: I read this as having the fireman 4 meters up from the floor. The f.b.d in the text's Solution Manual put him 4 meters below the top, but they did the problem as though he was 4 meters from the bottom. I think my way probably better reflects what you tried to do, assuming you read the problem and worked from there. In any case, we need to identify what the forces look like in the system, so we will start with a f.b.d. Assuming the ladder's weight acts at its *center of mass*:





We are going to start by using the translational version of N.S.L., noting that the *acceleration* in the system is zero and all of the variables are presented as magnitudes (signs unembedded).

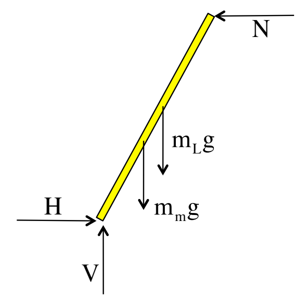


$$\sum F_x :$$
$$H - N = m\cancel{a}_x^{\,0}$$
$$\Rightarrow \quad H = N$$

$$\sum F_y :$$
$$V - m_L g - m_m g = m\cancel{a}_y^{\,0}$$
$$\Rightarrow \quad V = m_L g + m_m g$$
$$= (500.\ \text{Nt}) + (800.\ \text{Nt})$$
$$= (1.30\text{x}10^3\ \text{Nt})$$

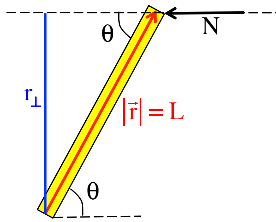
Two equations, three unknowns. We will get the last equation by summing the torques about whatever axis we choose, and putting that equal to zero (as the *angular acceleration* is zero in this case).

There is a subtlety I probably should point out at this time. Let's say you decided to sum the torques about the ladder's *center of mass*. The ladder's weight would produce no torque about that point so you'd have to generate a torque calculation for the force produced by *the man*, by *H*, by *V* and by *N*. In other words, you'd end up with an equation that had four pieces to it with three of those pieces being unknowns . Being clever, though, would have made life a lot easier for you. Specifically, if you took the torque about the contact point with the floor, *H* and *V* would produce no torque as they'd pass through the point about which you were taking the torque, you'd only have to generate three torque expressions, and you'd only have *one unknown* in the resulting equation. THIS IS THE WAY WE WILL GO!

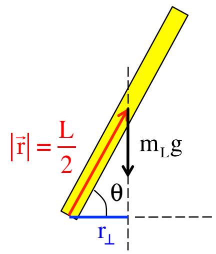


As a review, I'll lay out the process for taking torques as we go. I'll start by looking at the force and associated torque due to N.

Look at the ladder with just N being applied (for ease of viewing): Notice how $\vec{r}$ is identified in the sketch, and how $r_\perp$ (the shortest distance between the *line of N* and the point about which the torque is being taken) is identified. Note also how the angle is identified, and that $r_\perp = L\sin\theta$.



Doing similarly for the torque gravity exerts on the ladder (see second sketch) and we find that its $r_\perp$ is equal to $L/2\cos\theta$. (An expression for the man will look just like this, but the fraction will be different.)



Keeping track of signs and putting this all together, we can execute the torque calculations for all the forces that produce torques about the contact point with the floor, and in doing so generate our third equation.

Specifically:

$$\underline{\sum \Gamma_{floor}:}$$

$$-m_L g\left(\frac{L}{2}\cos\theta\right) - m_m g\left(d_m\cos\theta\right) + N\left(L\sin\theta\right) = I_{floor}\not{\alpha}^{\,0}$$

$$\Rightarrow \quad N = \frac{m_L g\left(\frac{L}{2}\cos\theta\right) + m_m g\left(d_m\cos\theta\right)}{L\sin\theta}$$

$$= \frac{(500.\text{ Nt})\left(\frac{(15.0\text{ m})}{2}\cos 60^\circ\right) + (800.\text{ Nt})\left((4.00\text{ m})\cos 60^\circ\right)}{(15.0\text{ m})\sin 60^\circ}$$

$$= 268 \text{ Nt}$$

We know that the magnitude of the horizontal floor force and the normal wall force are the same, so in summary we can write:

$$V = \left(1.30\text{x}10^3 \text{ Nt}\right) \quad \text{and} \quad H = \left(268 \text{ Nt}\right)$$

This is what the book's Solution Manual quotes, so I presume we are good!

b.) What is the coefficient of friction if the ladder breaks free when the fireman is 9 meters up the ladder?

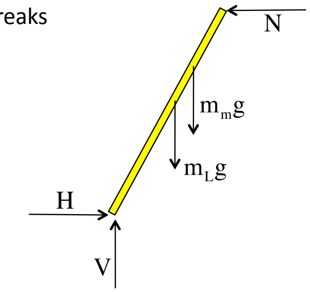
We know that

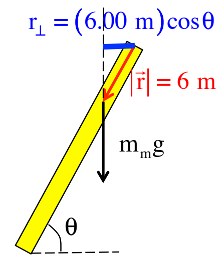
$$\mu_s = \frac{f}{V},$$

where $f = H = N$ (we derived the second part of this using N.S.L. in the first part of the problem).

The temptation might be to sum the torques about the wall contact (I started that process by figuring out $r_\perp$ for the man), but upon reflection, really, all we need to do is redo the torque calculation we've already done but with the man 9.00 meters up the ladder. Why? Because $f = H = N$ (we derived the second part of this using N.S.L. in the first part of the problem), so all we really need is *N*.

Executing all that is shown on the next page.





Sooo . . .

$$\underline{\sum \Gamma_{floor}:}$$

$$-m_L g\left(\frac{L}{2}\cos\theta\right) - m_m g\left(d_m\cos\theta\right) + N\left(L\sin\theta\right) = I_{floor}\not{\alpha}^{\,0}$$

$$\Rightarrow \quad N = \frac{m_L g\left(\frac{L}{2}\cos\theta\right) + m_m g\left(d_m\cos\theta\right)}{L\sin\theta}$$

$$= \frac{(500.\text{ Nt})\left(\frac{(15.0\text{ m})}{2}\cos 60^\circ\right) + (800.\text{ Nt})\left((9.00\text{ m})\cos 60^\circ\right)}{(15.0\text{ m})\sin 60^\circ}$$

$$= 421 \text{ Nt}$$

Completing the problem:

$$\mu_s = \frac{N}{V} = \frac{(421\text{ N})}{\left(1.30\text{x}10^3\text{ N}\right)} = .324$$